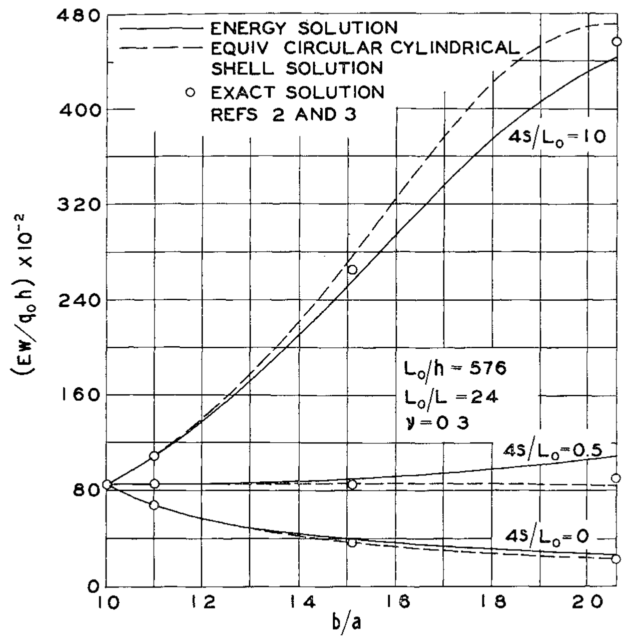

Fig 5 Radial displacements vs b/a for values of $4s/L_0$ at $2x/L = 0$

Donnell for circular, cylindrical shells if nonlinear (buckling) terms are omitted The stresses are related to the strains by Hooke's law for plane stress

Discussion

Numerical results have been obtained for oval shells in which $L_0/h = 576$ and $L_0/L = 24$ Poisson's ratio ν was taken as 0 3; Young's modulus E is arbitrary Figures 2–5 show the variation with b/a of the nondimensional membrane stress $(\sigma\ /q_0)_{z\ =\ 0}$, circumferential bending stress $\pm(\sigma_b/q_0)_{z\ =\ \pm h/2}$, axial bending stresses $\pm(\sigma_{xb}/q_0)_{z\ =\ \pm h/2}$, and radial displacement (Ew/q_0h) Each of these quantities is shown at mid-bay $(2x/L = 0)$ for the three generators $(4s/L_0) = 0$ (major axis), 0 5 (at which $r = r_0$), and 1 0 (minor axis)

For $b/a = 1\ 10$, 1 51, and 2 06, respectively, the maximum circumferential stress, a compressive stress given by $[\sigma_s/q_0]_{\max} = [(\sigma\ /q_0)_{z\ =\ 0} + (\sigma_b/q_0)_{\ =\ -(h/2)}]_{\max}$, varies at most by $\frac{1}{10}\%$, 6%, and 11% from the exact solution For the same values of b/a the maximum values of the axial bending stress are in error by $\frac{1}{20}\%$, 4%, and 7% Similarly, the maximum radial displacements differ by $\frac{1}{10}\%$, 3%, and 3% for the exact solutions These results, together with those of Ref 1, suggest that the energy solution used can be applied with some degree of confidence to boundary conditions other than those considered herein and in Ref 1

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Range and Angle Prediction Tracking of Objects with Definable Trajectories

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Introduction

THE usual prediction-tracking equations employed in most radar systems are a form of a Taylor's series in the time variable This type of prediction-tracking equation is written as

$$X_i = X_{i0} + \dot{X}_{i0}(t - t_0) + \ddot{X}_{i0}(t - t_0)^2/2 +$$

where X_i is a position coordinate of interest at time t predicted from the position, velocity, and acceleration data obtained at time t_0 Since the third time derivative of the position coordinate (the jerk function) is difficult to evaluate from radar data, this type of prediction equation is usually truncated after the quadratic term As a result of truncation errors, the Taylor's series form of prediction is limited to a time interval of the order of a second or less In addition to the truncation errors of the prediction equation, the method of prediction by storing ground coordinates (Cartesian coordinates with the origin at the radar site) involves coordinate transformation errors In the case of the radar range variable, the coordinate transformation errors are of paramount significance The radar range is the most accurate data obtainable from a short-pulsed radar signal Relative to the radar range resolution, the angular resolution of the target's data is extremely crude Thus, by performing a coordinate transformation from the radar range-angle coordinates to the ground coordinates, the accuracy of the radar range is diluted by the errors in the angular data

As a consequence of the propagation of errors through coordinate transformation, the natural solution is to attempt to store data and predict the target's behavior in the radar range-angle coordinate system Since the Taylor's series time predictor is inadequate for the forementioned reasons, a new approach was investigated for a completely general range-angle predictor The results of the analysis are applicable to ballistic missile trajectories, satellite trajectories, nonmaneuvering aircrafts, and any trajectory that lies in a given plane and whose motion in that plane is predictable or obeys a set of physical laws

The radar range tracking equation is an exact closed-form solution for the radar range in terms of the target parameters and is independent of the radar angle requirements for range-only tracking Angle tracking equations are also given in terms of the radar range and the target parameters

Assumptions

In the derivation of the range prediction-tracking equation, the analysis is based upon a spherical earth geometry The approximation is valid within the specified radar coverage volume since, for any given radar site, the oblateness effects can be taken into account by matching a spherical earth model to the known direction and magnitude of the radar site gravity vector Furthermore, the analysis is based upon a radar tracking station located on the surface of a nonrotating earth For practical applications of a radar site on a rotating earth at any given latitude, equations are easily generated for the rotating earth corrections

No assumptions are made with respect to the method in which the trajectory is defined, with the exception that it is

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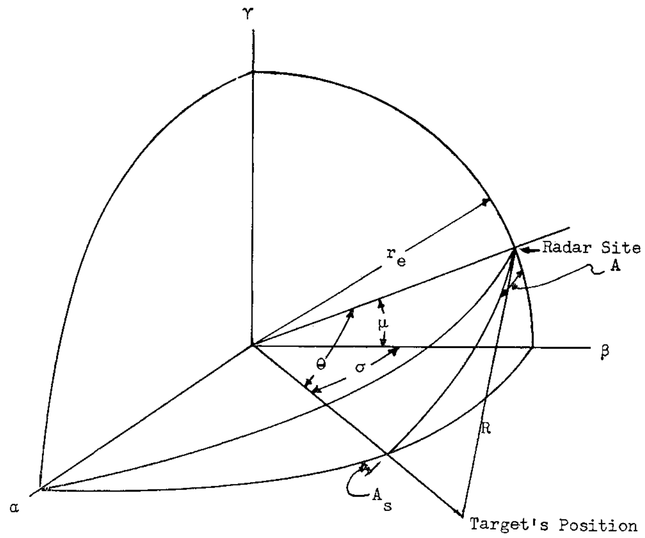

Fig 1 Radar-target geometry on a spherical nonrotating earth

definable either analytically or numerically and lies in a "great circle plane" Kumagai[1] derived the analytical expression for the second-order cross-range perturbation of satellite trajectories caused by the oblateness effect From the solution, it can be shown that the cross-range variation for satellite trajectories of less than 500 miles alt is less than 3 mrad during the passage of the satellite through the radar coverage volume Thus, the resulting error caused by neglecting the oblateness effect during the tracking time is of the order of (or smaller than) the angular resolution of a high-resolution pencil beam radar

The last assumption is that the radar signal propagation is along straight lines Although no attempt is made in this paper, the tracking equations can be modified to approximate the effect of the atmospheric refraction characteristics of the radar propagation If the tracking volume coverage is limited to elevation angles greater than 30°, neglecting the refraction characteristics introduces only small errors

Analysis

Radar range tracking equation

The derivation of the radar range prediction equation is based upon the differential equation of motion which relates the radar range to the components of the target velocity in the target trajectory plane (The target trajectory plane is defined as the plane containing the target's velocity and geocentric radius vectors)

The coordinate system that has been applied to the analysis is described in Fig 1 Referring to Fig 1, the α, β, γ axes are the inertial Cartesian coordinate system for a nonrotating earth model in which the α,β plane coincides with the target trajectory plane The radar site is located in the β,γ plane Relatively to this coordinate geometry, let

μ = geocentric angle between the radar site radius vector $\mathbf{r}$ and the trajectory plane measured in the β,γ plane
R = radar range to the target position
r = geocentric radius of the target position
r = effective radius of the earth in the local vicinity of the radar site
θ = geocentric angle between the radar site radius vector $\mathbf{r}$ and the target's radius vector $\mathbf{r}$
ϕ = radar elevation of the target's position relative to the radar site's local horizontal plane
A = angle between the earth-centered plane defined by the radar range vector and the trajectory plane measured in the target's local horizontal plane
σ = geocentric angle of the target's radius vector relative to the β axis
$\mathbf{e}_R$ = unit vector directed outwardly along the radar range vector
$\mathbf{e}$ = unit vector directed along the target's geocentric radius vector
$\mathbf{e}_{r\perp}$ = unit vector that is perpendicular to $\mathbf{e}$ and lies in the plane defined by $\mathbf{e}_R$ and $\mathbf{e}$
V_σ = component of the target's velocity along the target's local horizontal
$\mathbf{e}_\sigma$ = unit vector along the target's local horizontal direction

The radar range rate relative to the target's motion is the component of the target's velocity vector along the direction of the radar range vector, or

$$\dot{R} = \mathbf{e}_R \mathbf{V} \tag{1}$$

where the dot over a parameter indicates a time derivative When written in terms of the components of the velocity vector $\mathbf{V}$, Eq (1) becomes

$$\dot{R} = \mathbf{e}_R (\mathbf{e}\,\dot{r} + \mathbf{e}_\sigma V_\sigma) \tag{2}$$

The unit radar range vector $\mathbf{e}_R$ can be expressed as a linear combination of the two orthogonal unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\perp$ as (Fig 2)

$$\mathbf{e}_R = \mathbf{e}\ \sin(\theta + \phi) + e_{r\perp} \cos(\theta + \phi) \tag{3}$$

Upon substituting Eq (3) into Eq (2),

$$\dot{R} = \dot{r} \sin(\theta + \phi) + \mathbf{e}_\perp\ \mathbf{e}_\sigma V_\sigma \cos(\theta + \phi) \tag{4}$$

$$= \dot{r} \sin(\theta + \phi) + V_\sigma \cos A\ \cos(\theta + \phi) \tag{5}$$

The following can be derived from spherical trigonometric relationships:

$$\cos A = \cos\mu \sin\sigma / \sin\theta$$

Substituting this relationship in Eq (5),

$$\dot{R} = \dot{r} \sin(\theta + \phi) + [V_\sigma \cos(\theta + \phi)/\sin\theta] \cos\mu \sin\sigma \tag{6}$$

From the geometry of Fig 2,

$$\frac{\cos(\theta + \phi)}{\sin\theta} = \frac{(r_e \cos\phi)/r}{(R \cos\phi)/r} = r_e/R$$

$$\sin(\theta + \phi) = \frac{R + r_e \sin\phi}{r}$$

With these relationships, Eq (6) becomes

$$\dot{R} = (\dot{r}/r)(R + r_e \sin\phi) + V_\sigma (r_e/R) \cos\mu \sin\sigma \tag{7}$$

Upon multiplying through by R and substituting $V_\sigma = r\dot{\sigma}$, Eq (7) becomes

$$R\dot{R} = (\dot{r}/r)(R^2 + r\,R \sin\phi) + r\,r\dot{\sigma} \cos\mu \sin\sigma \tag{8}$$

The local radar zenith component of the target position can be expressed in terms of the relationship between the h (height) to $Z(= R \sin\phi)$ conversion as

$$Z = h + \frac{h^2 - R^2}{2r_e} = \frac{r^2 - r_e^2 - R^2}{2r_e} \tag{9}$$

Substituting Eq (9) in Eq (8) results in an integrable differential equation relating the radar range rate to the target's radar range and position-velocity parameters as

$$\frac{d}{dt}\frac{(R^2)}{2} - \frac{\dot r}{r}\frac{R^2}{2} = \frac{\dot r}{r}\frac{(r^2 - r_e^2)}{2} + r\,r\dot\sigma\,\cos\mu\,\sin\sigma \qquad (10)$$

The integrating factor for Eq (10) is $\exp[-\ln(r/r_0)] = (r_0/r)$

After multiplying both sides of Eq (10) by the integrating factor, integrating between initial and final values, and solving for R^2, there results

$$\frac{R^2}{2} = \frac{R_0^2}{2}\frac{r}{r_0} + \frac{r_0 - r}{2}\left(\frac{r_e^2}{r_0} - r\right) + r r_e \cos\mu(\cos\sigma_0 - \cos\sigma) \qquad (11)$$

where the subscript zero indicates the initial conditions of the integration interval Equation (11) is the desired range equation in its general form based upon a nonrotating earth in order to maintain the parameter $\cos\mu$ equal to a constant The form of Eq (11) is applicable to any analytically definable trajectory that is confined to a prescribed great circle plane Examples are satellite and ballistic missile trajectories that are definable by the Keplerian equations during the target's passage through the radar surveillance volume, constant flight path aircraft trajectories in which r = const and $\sigma = Vt/r$, earth-moon trajectories, and guided missile trajectories with pitch control only The only requirement for its application is that the earth-centered radius r of the trajectory be known as a function of time or σ

Elevation angle tracking equation

Using the results of the radar range tracking equation, a simple analysis leads to the elevation angle tracking equation Referring to Fig 2, let

X_g = radar ground range to the target
Z = radar zenith component of the radar range

Then

$$X_g = R\cos\phi$$
$$Z = R\sin\phi \qquad (12)$$
$$X_g^2 + (r_e + Z)^2 = r^2$$

Upon taking the total differential of Eq (12) and dividing each term by the product $(r_eR)dt$, the following first-order differential equation results:

$$\frac{d(\sin\phi)}{dt} + \frac{(dR/dt)}{R}\sin\phi = \frac{r(dr/dt)}{r_eR} - \frac{(dR/dt)}{r_e} \qquad (13)$$

The integrating factor for Eq (13) is $\exp[\ln(R/R_0)] = R/R_0$

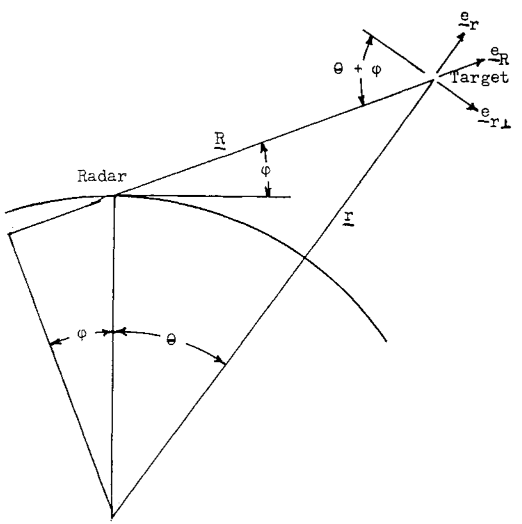

Fig 2 Earth-centered geometry in the plane of the radar site and target positions

Thus, integrating Eq (13) between the initial and final conditions, a form of the elevation angle updating equation is

$$\sin\phi = \frac{R_0}{R}\sin\phi_0 + \frac{r^2 - r_0^2}{2r\,R} + \frac{R_0^2 - R^2}{2r\,R} \qquad (14)$$

An equivalent form of Eq (14) can be derived from the complete h (height) to Z conversion:

$$Z = h + (h^2 - R^2)/2r \qquad (15)$$

Multiplying Eq (15) through by $2r$ and completing the square results in

$$\sin\phi = (r^2 - r_e^2 - R^2)/2r_eR \qquad (16)$$

Equation (16) is equivalent to Eq (14)

Azimuth tracking equation

Referring to the geometry of Fig 1, let the radar Cartesian coordinate system be defined by the X, Y, and Z axes in which

Z = zenith axis
Y = axis that lies in the β,γ plane directed toward the target plane
X = axis that is parallel to the target plane defined by a right-handed coordinate system

In addition, let

A = azimuth angle of the radar range vector relative to the Y axis

Then, from spherical trigonometric relationships, it can be shown that

$$\sin A = \sin\sigma/\sin\theta = r\sin\sigma/R\cos\phi$$

In terms of the direction cosine angle ξ relative to the Y axis, $\sin\xi = r\sin\sigma/R$ from the relationship $\sin\xi = \sin A\cos\phi$

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Density Distribution over a Moving Circular Plate in Free-Molecule Flow (*U*)

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The free-molecule flow field is calculated on the front side of a moving circular plate including the effect of finite area distribution on the density gradient of the reflected molecules Calculations are carried out both for diffuse and specular reflections, and the results are compared with those obtained from differential area considerations

BIRD[1] made a thorough analysis of the free-molecule flow field around moving bodies of different geometries with surface elements located at the origin To study the effect of finite distribution of surface elements on this flow field on the plate axis as well as on the other arbitrary locations, we start with Bird's analysis

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